

# Development of a Modified One-Strut Design Model for Shear Strength of Masonry Infilled Frames with Opening

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**Abstract:** In this paper, the results of a study on the effect of brick masonry infill panels with centrally located openings on the performance of reinforced concrete frames under lateral loading is reported. An equivalent diagonal strut was used to model the stiffening effect of the masonry panels. A strength reduction factor was developed as a function of the opening ratio in the form of an exponential function and incorporated in the one-strut model. The study was also extended to finite element micro modeling on structural specimens of infill panels with varying opening ratios of 10%, 20%, 30%, 40% and 50% in order to validate the modified one-strut model. The practical agreement of the outputs of the modified one-strut model with those of the FE model confirms the adequacy of the proposed model for prediction and design of the shear strength of infilled frame under lateral loading.

**Keywords:** Infilled Frame, Bare Frame, Modified One Strut Macro Model, FE Micro Model.

## 1.0 INTRODUCTION

The recent trend of building construction is characterized by massive application of reinforced concrete frames, infilled with walls for load bearing or partitioning purposes. In multistory frames, shear walls are used as bracing elements. These infill walls are usually structural masonry, made up of concrete or burnt clay bricks, bonded together in cement mortar. Structural masonry was traditionally widely used in civil and structural engineering works including buildings, tunnels, bridges, aqueducts, retaining walls, sewerage systems among others. At present, there has been growing interest by researchers to relate masonry work design to its actual behavior which has reflected in the increased research in this area [1-6].

From the first attempts to model the response of composite infilled frame structures, experimental and conceptual studies have suggested that a diagonal strut with the appropriate geometrical and material characteristics could be used to represent the composite action of an infilled frame. Several investigators [7-15] have proposed variations of the equivalent strut model, with the key parameter being the effective width of the strut. Polyakov [16], first studied the possibility of considering the effect of the infill panel as equivalent to diagonal bracing and this suggestion was later taken up by Holmes [7], who replaced

the infill by an equivalent pin-jointed diagonal strut, made of the same material and having the same thickness as the infill panel and a width equal to one third of the infill diagonal length. Some other researchers related the width of the equivalent diagonal strut to the infill/frame contact lengths using an analytical solution, adapted from the equation of beam on an elastic foundation, subjected to a concentrated load.

Holmes [7] suggested that the effective width of an equivalent strut depends primarily upon the thickness and aspect ratio of the infill. Stafford-Smith and Carter [17] have posited that the equivalent strut width is not a constant value, but varies with the applied loading and the relative properties of the frame and infill. However, Mehrabiet al. [18] found that the lateral stiffness of the infilled frames using Stafford-Smith and Carter's equivalent struts is consistently underestimated by a factor of two when the bending stiffness for uncracked RC sections is used. Further attempts to capture the interaction of in-plane and out-of-plane strength under bi-directional loading have resulted in the introduction of three-strut and multiple-strut models [12], [15], [19], [20], [21], [22].

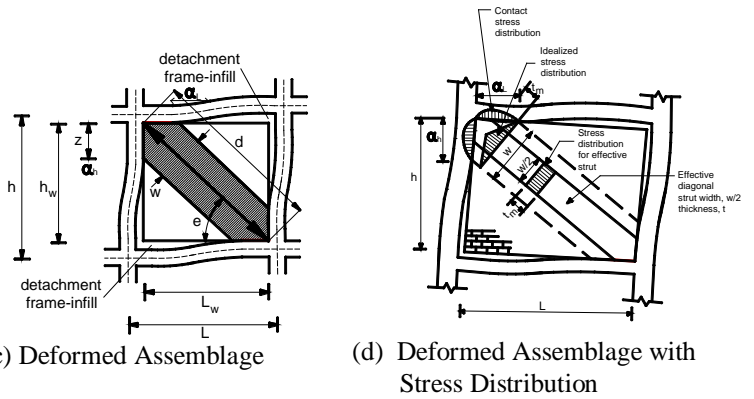
Infill walls in frames frequently contain door and window openings at the different locations, which naturally reduces stiffness and load carrying capacity of the diagonal strut depending upon the size of opening and its location.

Experimental and analytical studies by Benjamin and Williams [23] show that centrally located openings may reduce the stiffness and strength of diagonal strut by about 75% and 40% respectively. In spite of these facts, most researches have tended to concentrate on simple cases of infill wall without openings. Analytical study of infill frame with openings is limited and has little comparison due to the differences in the type of materials used and openings considered.

From the above, it can be seen that the consideration of the infill panel in the design of RC frame structures results in a complex modeling problem because of the large number of interacting parameters and the many possible modes of failure that need to be evaluated with a high degree of uncertainty. The need to obtain deeper understanding of the influence of openings on the composite behavior of infilled frames has further led to development of more and more complex models with ever increasing number of parameters. An experimental study in which all these factors could be taken into account is difficult to implement for obvious reasons. Thus, in most cases the use of finite element approach has been considered a most viable option in spite its computational complexities and resource requirements. For these reasons, the need for more simplified models of the composite behavior of infilled frame has been recognized by researchers. In this paper, a single diagonal strut model, capable of predicting the shear strength response of infilled frames with openings is presented. A unique feature, enhancing the efficiency of the model, is the introduction of a strength reduction factor, developed by the authors in the study.

**2.0 PROPOSED DESIGN MODEL FOR MASONRY INFILLED FRAME**

Experimental evidence have shown that under racky loads, the infill tends to separate at the unloaded corners while maintaining clearly defined contact zones with the frame at the loaded corners. Diagonal cracks are observed along the compression diagonal. This lends credence to the assumption that the infill panel could be replaced by an equivalent pin-jointed diagonal compression strut of the same material and having the same thickness as the infill panel. The mechanism of response of a typical infilled frame is shown in Figure 1.



**Figure 1: Structural Mechanism of Infilled Frame and Equivalent One-Strut Model**

**2.1 Shear Strength Reduction Factor**

Studies by Hendry [16] have shown that the geometric properties of the diagonal strut are functions of the length of contact  $\alpha_h$  and  $\alpha_L$  between the wall and the column and the beam respectively (Figure 1). Hence, assuming a beam on elastic foundation, the following expressions are obtained for the contact lengths [17]:

$$\alpha_h = \frac{\pi}{2} \sqrt[4]{\frac{4E_f I_c h}{E_m t \sin 2\theta}} \quad (1)$$

$$\alpha_L = \pi \sqrt[4]{\frac{4E_f I_b L}{E_m t \sin 2\theta}} \quad (2)$$

- Where  $E_m, E_f$  = elastic moduli of the masonry wall and frame material respectively.
- $t, h, L$  = thickness, height and length of the infill wall, respectively.
- $I_c, I_b$  = moments of inertia of the column and the beam of the frame respectively.
- $\theta$  =  $\tan^{-1}h/L$

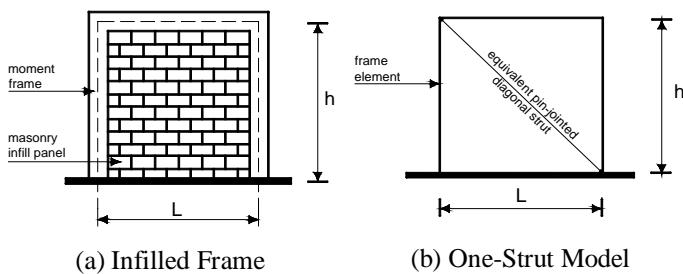
Also from Figure 1, assuming a triangular stress distribution along the width of strut  $w$ , the force in the strut is  $1/2 f_m w t$ , where the average compressive stress is one-half of the maximum stress  $f_m$ .

Using the Pythagoras equation, the effective strut width of infill without opening equals

$$w = \sqrt{\alpha_c^2 + \alpha_h^2} \quad (3)$$

The shear bracing capacity of the strut will be reduced by the presence of openings by a factor which will depend on the opening ratio ( $\beta$ ), expressed as the ratio of the opening area to the area of the solid infill panel. Now, assuming  $w_o$  is the effective width of the diagonal strut with opening and  $f_m$  the compressive strength of the masonry, the infill strength reduction factor can be obtained as follows.

Resistance of solid infill  $R = 1/2 f_m w t = 1/2 f_m A$   
 Resistance of infill with opening



$$R_o = \frac{1}{2} f_m w_o t = \frac{1}{2} f_m A_m$$

Stress reduction factor due to openings

$$\lambda_m = R_o/R = A_m/A$$

Hence

$$A_o = \lambda_m A \tag{4}$$

In order to modify the equivalent diagonal area to account for openings, a suitable equation was obtained by regression analysis on data obtained from previous experimental and analytical works by the authors, where the shear strength reduction factor ( $\lambda_m$ ) was related to the opening ratio ( $\beta$ ) of the infill panel in the form

$$\lambda_m = e^{0.06\beta} \tag{5}$$

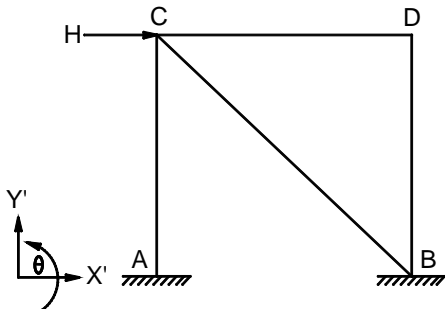
### 2.2 Global Stiffness Matrix for One Strut Model

Once the geometric and material properties of the strut are established, the analysis of the infilled frame was carried out using the stiffness matrix method in which the diagonal strut is modeled as a pin-jointed bar element while the frame members were modeled as rigid jointed members. Analyzing the equivalent frame using classical methods of structural analysis in the matrix stiffness method for a frame structure, maximum unknown horizontal deflections were obtained from the solution of the global structural matrix where the force vector and the displacement vector was related as in equation 6

$$F = [K] [\delta] \tag{6}$$

Where F and  $\delta$  represents the force and displacement vectors considering two degrees of freedom at each node.

To assemble the global stiffness matrix K, consider the equilibrium of the one strut model in Figure 2.



**Figure2: One-Strut Model for Macro-Modeling of Infilled Frame Structure**

The relevant degrees of freedom (DOF)

$$d = d_A, d_B, d_C, d_D$$

The corresponding load vector

$$P = P_A, P_B, P_C, P_D$$

The equilibrium equation takes the form

$$\begin{bmatrix} P_A \\ P_B \\ P_C \\ P_D \end{bmatrix} = \begin{bmatrix} k_{11(AC)} & 0 & k_{12(AC)} & 0 \\ 0 & k_{11(BD)} + k_{11(BC)} & k_{12(BC)} & k_{12(BD)} \\ k_{21(AC)} & k_{21(BC)} & k_{22(AC)} + k_{22(BC)} + k_{11(CD)} & k_{12(CD)} \\ 0 & k_{21(BD)} & k_{21(CD)} & k_{22(CD)} + k_{22(BD)} \end{bmatrix} \begin{bmatrix} d_A \\ d_B \\ d_C \\ d_D \end{bmatrix} \tag{7}$$

The component stiffness matrix for the pin-jointed diagonal strut BC, is given by

$$[k_{BC}] = \frac{EA_m}{L} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ m^2 & -lm & -m^2 & \\ & l^2 & lm & \\ \text{Symmetrical} & & m^2 & \end{bmatrix} \tag{8}$$

Where E and  $A_m$  represents the elastic modulus of masonry and modified area respectively.

The component stiffness matrices for the plane frame elements AC, CD and BD were obtained using the general stiffness matrix equation. For example the transformed element stiffness  $k_{11}$  equals

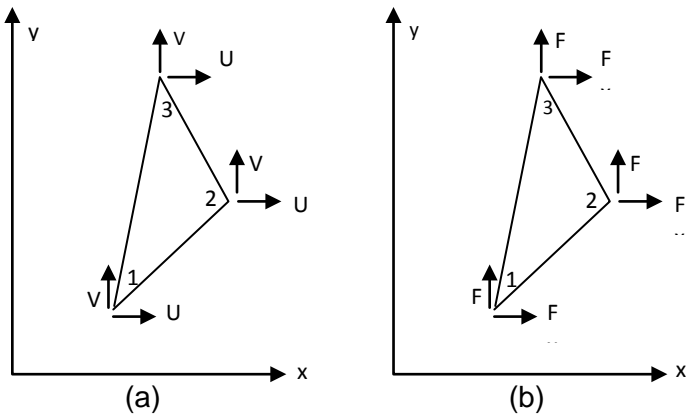
$$k_{11} = \frac{EI}{L} \begin{bmatrix} \frac{A}{I} l^2 + \frac{12}{L^2} m^2 & lm \left( \frac{AL}{I} - \frac{12}{L^2} \right) & -\frac{6}{L} m \\ \frac{AL}{I} m^2 + \frac{12}{L^2} l^2 & \frac{6}{L} l \\ \text{Symmetric} & 4 \end{bmatrix} \tag{9}$$

The structural or global stiffness matrix K is obtained by summing up the contribution from the 4 elements using standard structural mechanics technique.

### 3.0 VALIDATION OF THE MODEL

The model was validated by comparing the deflection profile, obtained from the outputs of the one-strut model and those from the FE model based of the constant strain plane triangular element. The basic concepts of the finite element method are well documented [24-26]. Hence only the essential features of the model will be presented here.

For this analysis, a three-node triangular finite element model with two degree of freedom (DOF) at each node is presented in Figure 3



**Figure 3: (a) Nodal Displacements Vectors (b) Nodal Force Vectors, Displayed in the Cartesian Co-Ordinate System.**

The major assumptions of modeling masonry behavior under plane stress include

- i the material is homogenous and elastic and hence obeys Hooke's law
- ii the displacement can be approximated by the polynomials

$$\begin{aligned} u(x, y) &= \alpha_1 + \alpha_2 x + \alpha_3 y \\ v(D, y) &= \alpha_4 + \alpha_5 x + \alpha_6 y \end{aligned} \quad (10)$$

With these assumptions, the element stiffness of the constant strain triangle has been established in the form

$$\{k^e\} = \left\{ \int [B]^T [D] [B] d(vol) \right\} \quad (11)$$

where B is the strain vector, D the elasticity matrix.

Substituting the volume of the triangular element, the equilibrium equation for the analysis of a typical triangular element becomes

$$\{F^e\} = \left[ [B]^T [D] [B] \Delta t \right] \{\delta^e\} \quad (12)$$

Where  $\Delta$  represents area of triangular element and  $t$  represents thickness.

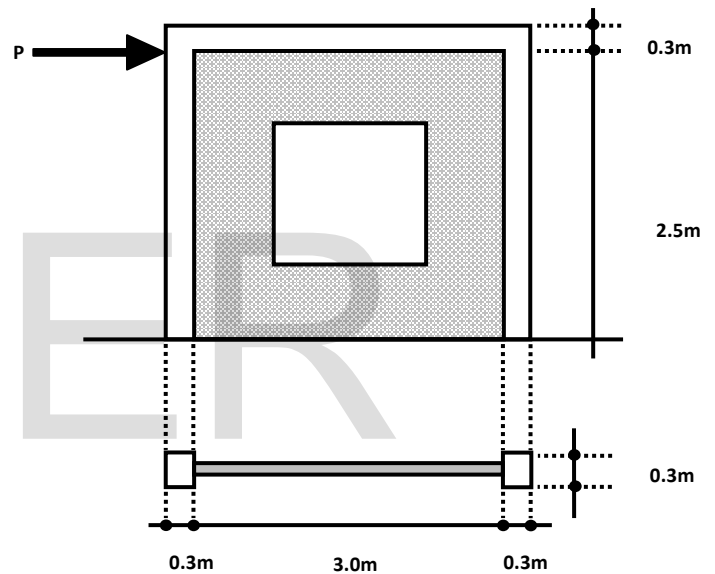
### 3.1 The Finite Element Computer Program

The FE model used in this paper is supported by a visual basic program developed by the authors. The computer program is divided into two parts. The first part consists of the routines for the control numbers and data input modules, the second part consists of routines for tabulated output of nodal displacements and element stresses. The basic steps to obtain the element stiffness matrix  $[K^e]$  and stress matrix  $[H]$  have already been discussed in details in the previous section and would involve voluminous numerical work, hence these processes were well built up in

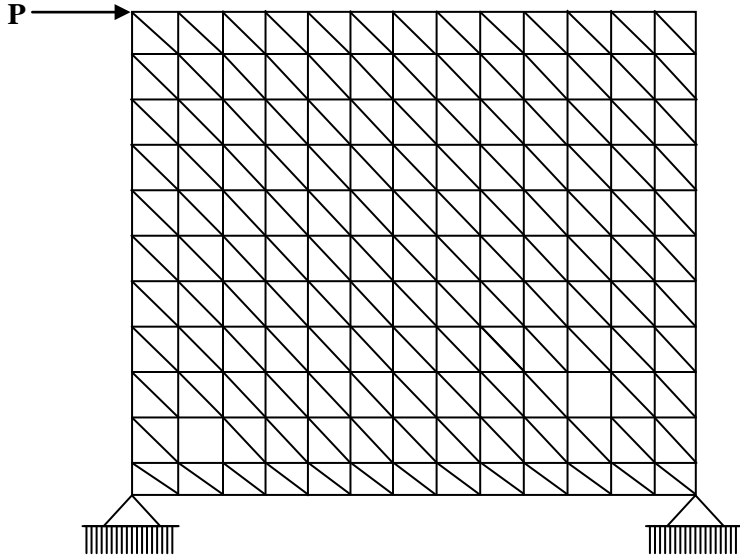
the subroutines to take care of the overall analysis. The input data consists of specifying the geometry of the idealized structure, its mechanical properties, the loading and the support condition. The data also includes certain control numbers that would help the efficiency of the program such as the total number of nodes and elements.

## 4.0 IMPLEMENTATION OF THE PROCEDURE

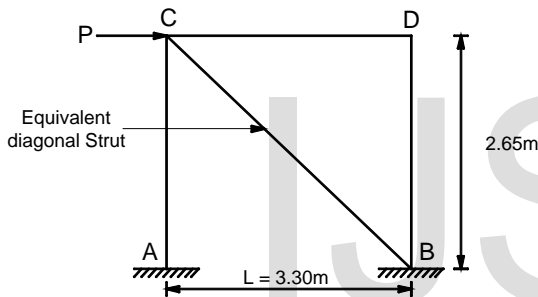
A typical representation of a single-bay single-storey masonry infilled RC frame (Figure 4) under lateral static load is subjected to analysis using the FE model and the one strut model and the opening in the infill panel varied from 0% to 50% for structural models tagged IF01, IF02, IF03, IF04 and IF05. The triangularly mesh structure ready for finite element analysis is shown in Figure 5, while a typical one strut model is seen in Figure 6



**Figure 4: Single-Storey Single-Bay Masonry Infilled RC Structure with Central Opening**



**Figure 5: Triangulantly Meshed Micro-Model for Finite Element Analysis**



**Figure 6: One-Strut Modified Masonry Macro Model of Infilled Frame Structure**

## 5.0 RESULTS

The computed results of the lateral displacement of the infilled frame, obtained on the basis of the one-strut and the finite element models for various values of applied horizontal loads and opening ratios are shown in Table 1 and plotted in Figure 7.

From the Table and plots, it can be seen that, the lateral displacement generally increased as the opening ratio increased, indicating the importance of the infill panel to the lateral resistance of the infilled RC frame structure.

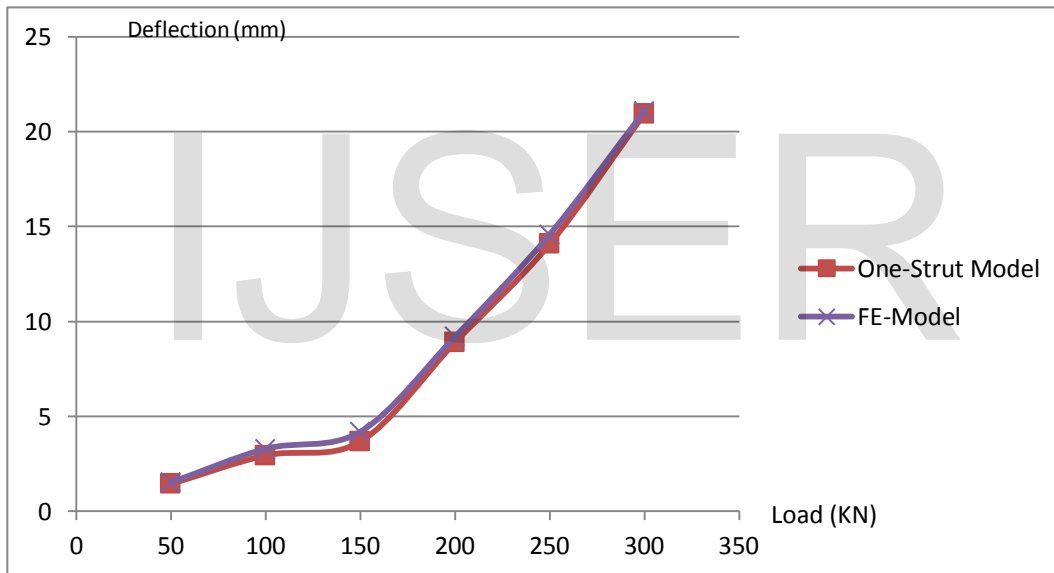
The functional dependence of lateral deflection with load for a given opening ratio is represented by two linear segments with bifurcation occurring approximately at a lateral load magnitude of 200 kN, after which the graph suffers a steep accent. Meanwhile, the variation of lateral deflection with opening ratio for a given load is much gentle in the range 8 - 12.5 mm; 13 - 23 mm and 21 - 32.5 mm for load values of 200, 250 and 300 kN, respectively.

The results confirm that the average error between the two models is about 3.95%, while the highest and least deviations of 4.53% and 3.28% occurred on structural models IF0 (10% opening) and IF05 (50% opening) respectively. Hence, there is a close agreement between the outputs of the proposed modified one-strut model and the FE model underscoring the adequacy of the proposed model to reproduce the response of infill frames including those with openings.

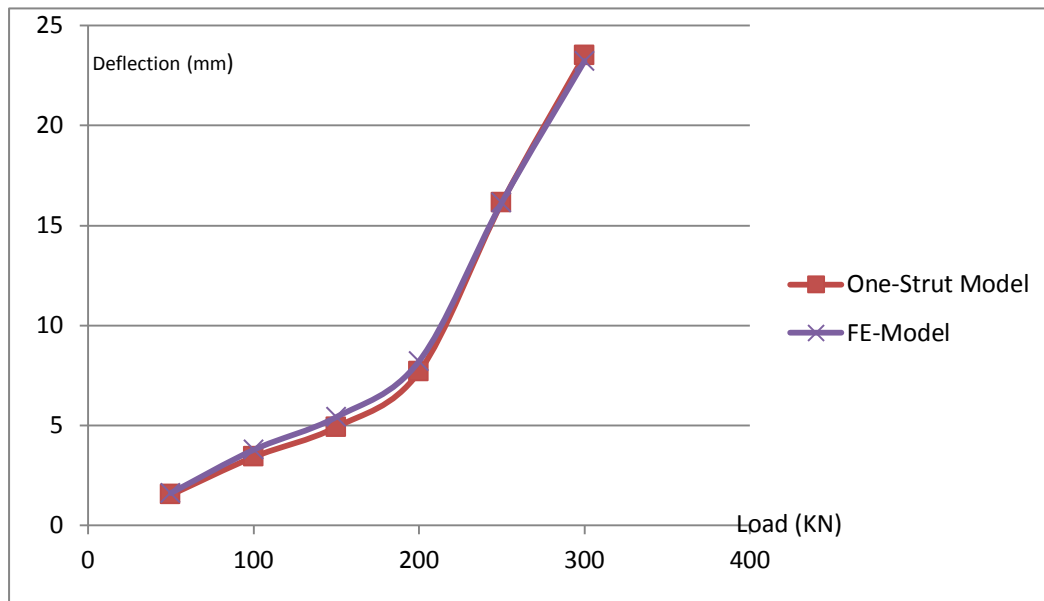
**Table 1: Computed Deflections of Infilled Frame for Various Loads and Opening Ratios**

| Specimen | Opening Ratio % | Model           | Deflection at different Load Application (mm) |       |       |       |       |       |
|----------|-----------------|-----------------|---|-------|-------|-------|-------|-------|
|          |                 |                 | 50kN  | 100kN | 150kN | 200kN | 250kN | 300kN |
| IF01     | 10%             | One Strut Model | 1.45  | 2.95  | 3.68  | 8.91  | 14.09 | 20.95 |
|          |                 | FE model        | 1.53  | 3.10  | 4.01  | 9.21  | 14.59 | 21.09 |
|          |                 | Diff. %         | 5.52  | 5.08  | 8.96  | 3.37  | 3.55  | 0.67  |
| IF02     | 20%             | One Strut Model | 1.53  | 3.42  | 4.92  | 7.68  | 16.13 | 23.51 |
|          |                 | FE model        | 1.62  | 3.58  | 5.12  | 8.01  | 16.15 | 23.22 |
|          |                 | Diff. %         | 5.88  | 4.68  | 4.06  | 4.30  | 0.12  | 1.23  |
| IF03     | 30%             | One Strut Model | 1.56  | 3.56  | 5.24  | 7.91  | 16.33 | 25.41 |
|          |                 | FE model        | 1.67  | 3.70  | 5.51  | 8.20  | 17.10 | 25.95 |
|          |                 | Diff. %         | 7.05  | 3.93  | 5.15  | 3.67  | 4.71  | 2.12  |
| IF04     | 40%             | One Strut Model | 1.91  | 4.61  | 8.10  | 10.74 | 17.89 | 26.01 |
|          |                 | FE model        | 1.99  | 4.51  | 7.72  | 11.02 | 19.11 | 27.12 |
|          |                 | Diff. %         | 4.19  | 2.17  | 4.69  | 2.61  | 6.81  | 4.27  |
| IF05     | 50%             | One Strut Model | 2.90  | 6.21  | 7.71  | 12.21 | 22.23 | 34.92 |
|          |                 | FE model        | 2.79  | 5.79  | 8.05  | 12.44 | 22.45 | 34.12 |
|          |                 | Diff. %         | 3.79  | 6.76  | 4.41  | 1.88  | 0.54  | 2.29  |

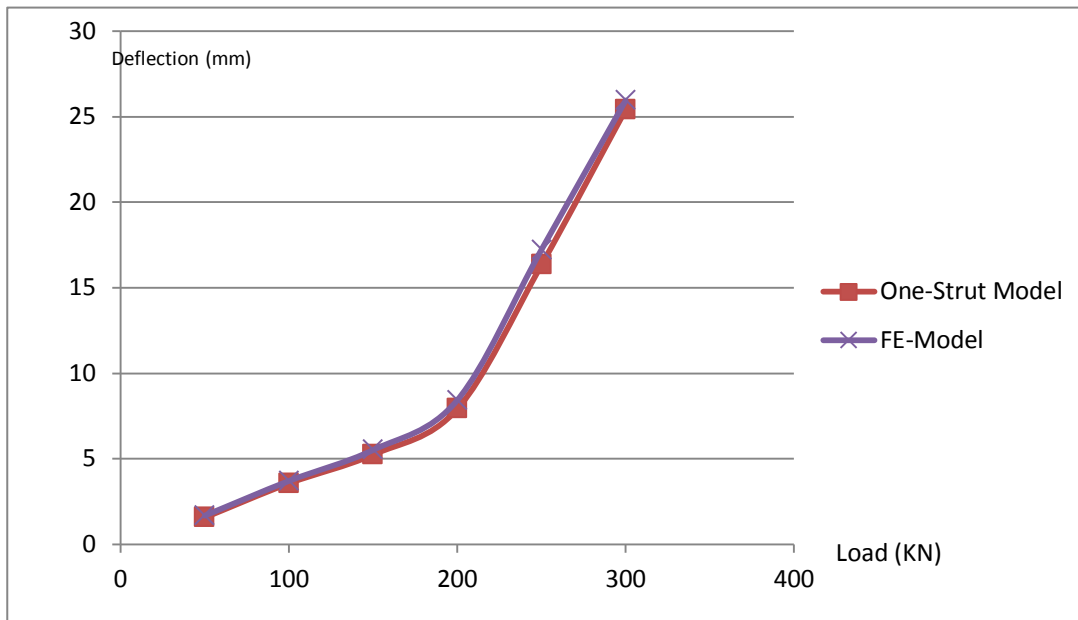
(a)



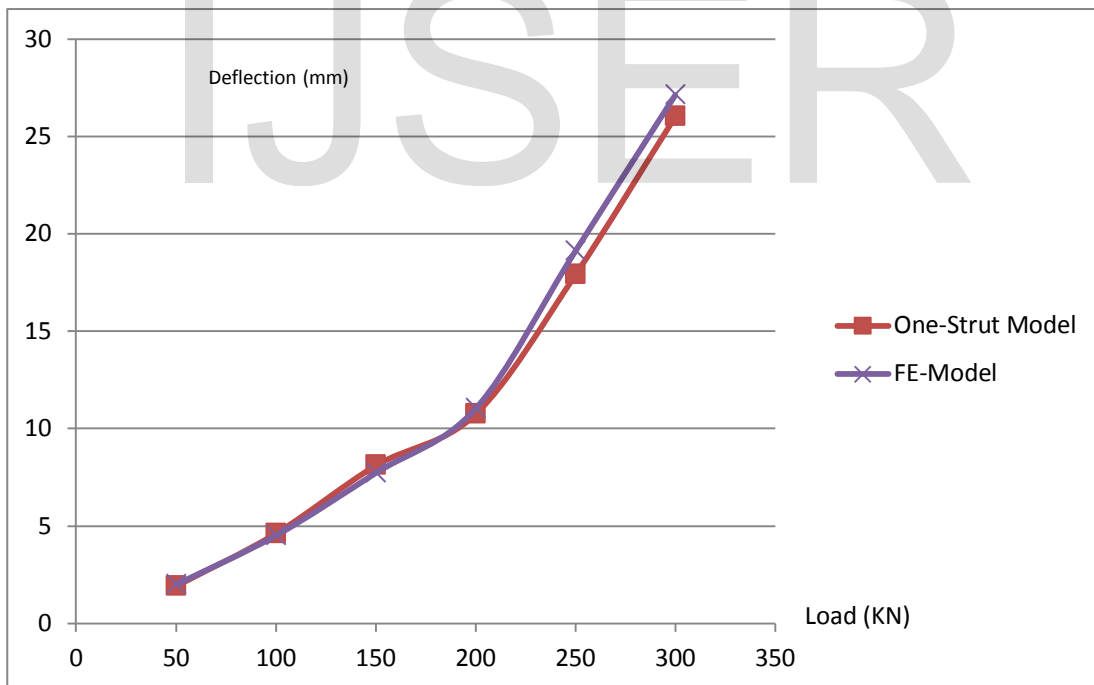
(b)

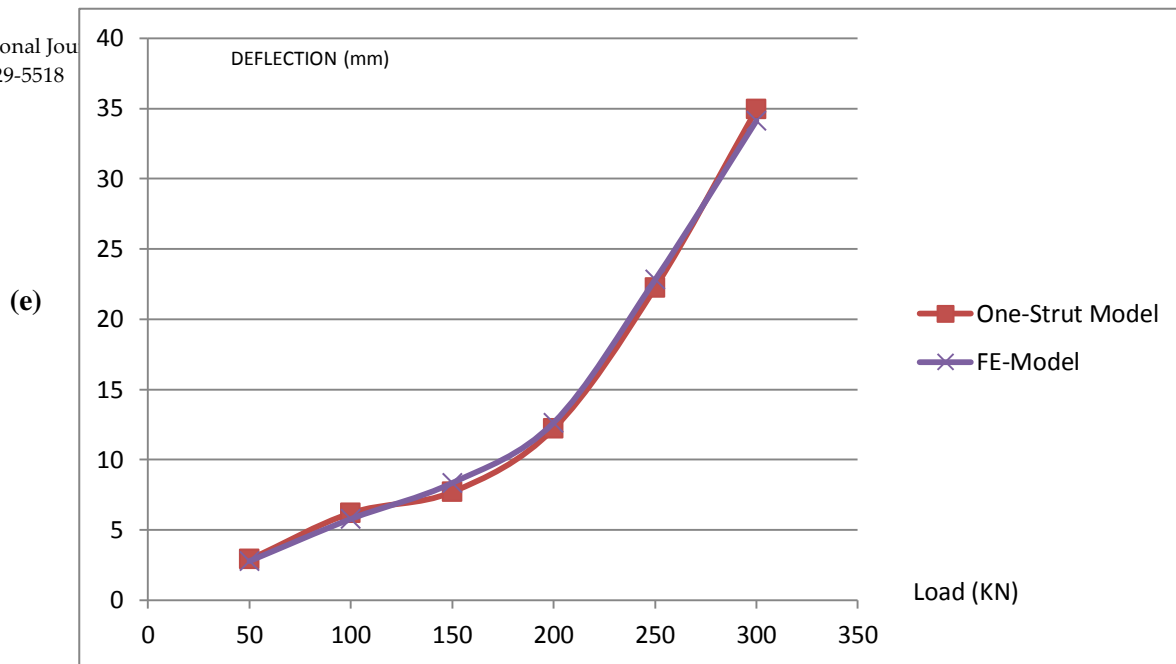


(c)



(d)





**Figure 7: Lateral Load-Deflection Profile for Different Size of Opening of Infilled Frame (a) IF01, (b) IF02, (c) IF03, (d) IF04, (e) IF05**

#### 4.0 CONCLUSION

In this paper, the possibility of modeling an infilled frame by modifying the pin-jointed single diagonal strut that replaces the brick infill is carried out. The ability of the modified diagonal area to appropriately model the effect of the varying opening sizes in the infill panel was established, resulting in close agreement of the model outputs with those from FE analysis.

The results confirm that the average error between the two models is about 3.95%, while the highest and least deviations of 4.53% and 3.28% occurred on structural models IF0 (10% opening) and IF05 (50% opening) respectively. Hence, there is a close agreement between the outputs of the proposed modified one-strut model and the FE model underscoring the adequacy of the proposed model to reproduce the response of infill frames including those with openings.

From the results the following specific conclusions can be made

1. Opening naturally lead to reduction in the shear strength of the infill. The strength reduction factor due to opening

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varies exponentially with the opening ratio and had the form  $\lambda_m = e^{0.06\beta}$

2. The lateral displacement generally increased as the opening ratio increased, indicating the importance of the infill panel to the lateral resistance of the infilled RC frame structure.

3. A unique value of lateral load of about 200kN, at which the lateral deflection changes from gradual growth to rapid variation, was established for all opening ratios considered.

4. The variation of lateral deflection with opening ratio for a given load is much gentle in the range 8 - 12.5 mm; 13 - 23 mm and 21 - 32.5 mm for load values of 200, 250 and 300 kN, respectively.

The FE model used in this paper has already been validated in previous work by the authors [27]. However some important factors require further investigating. The consistency of the regression equation obtained for modification of the area of diagonal strut to different type of infill material is recommended to be considered. Furthermore a more generalized regression equation to account for different configuration and position of opening is recommended.

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